5. Edge Detection, Image Segmentation and Mathematical Morphology

- Chapter 7 (Image Segmentation)
- Chapter 8 pp.518-528 (Morphology)

Edge Detection Using Derivative Operators

- **Edges:** the image portions which have large gradients
- Magnitude of the gradient

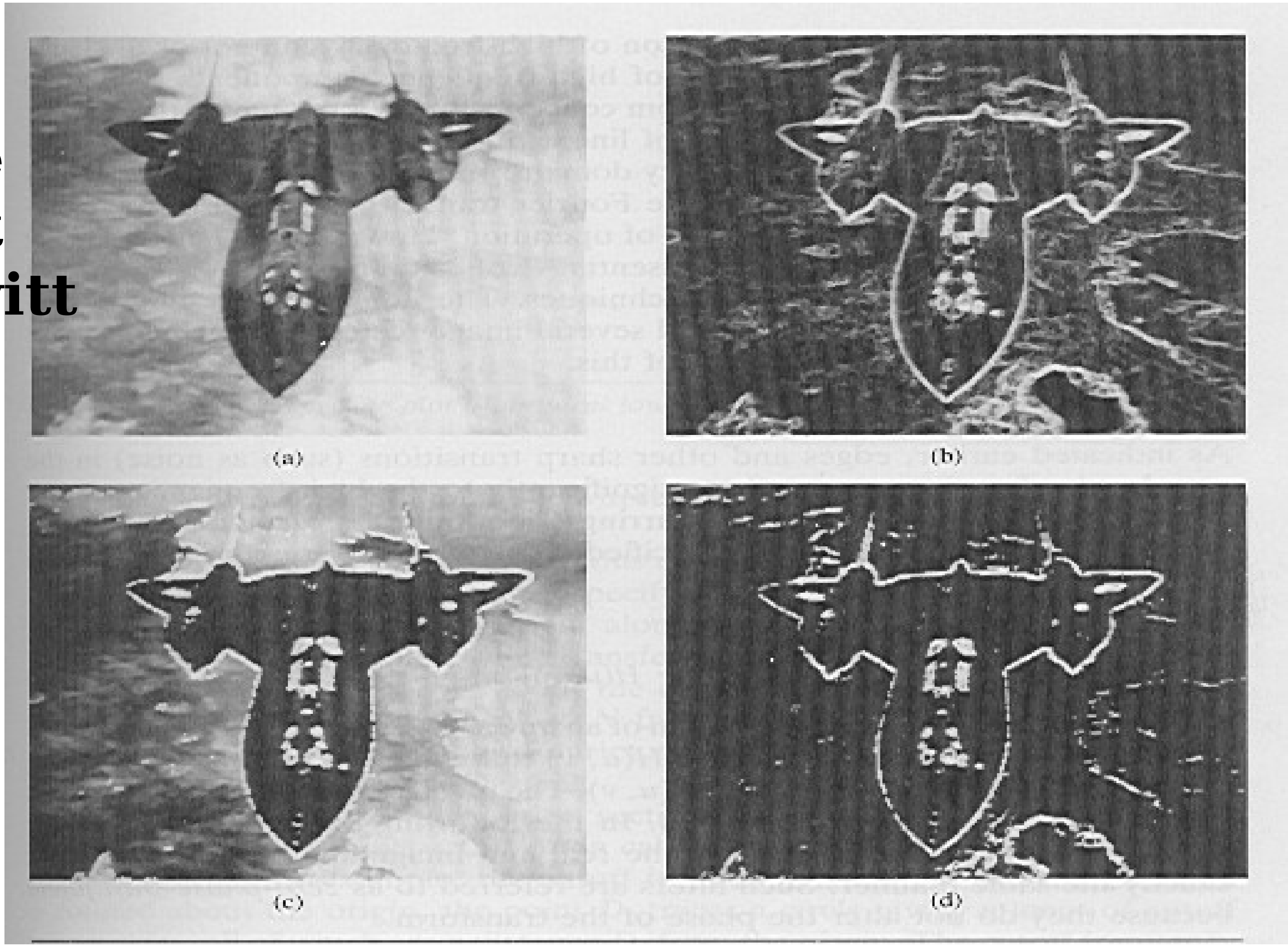
$$|\nabla f(n_1,n_2)| = \sqrt{\left[\frac{\partial f(n_1,n_2)}{\partial n_1}\right]^2 + \left[\frac{\partial f(n_1,n_2)}{\partial n_2}\right]^2}$$

often approximated by $\left|\frac{\partial f(n_1,n_2)}{\partial n_1}\right| + \left|\frac{\partial f(n_1,n_2)}{\partial n_2}\right|$

Thresholding $\Longrightarrow$ Edge Thinning & Grouping $\Longrightarrow$ Edge Map

Edge Enhancement by Gradients

a): original
b): magnitude
f the gradient
sing the Prewitt
perator
c): Gradient >
5 --> 255
d): (c) and
radient < 25
-> 0.



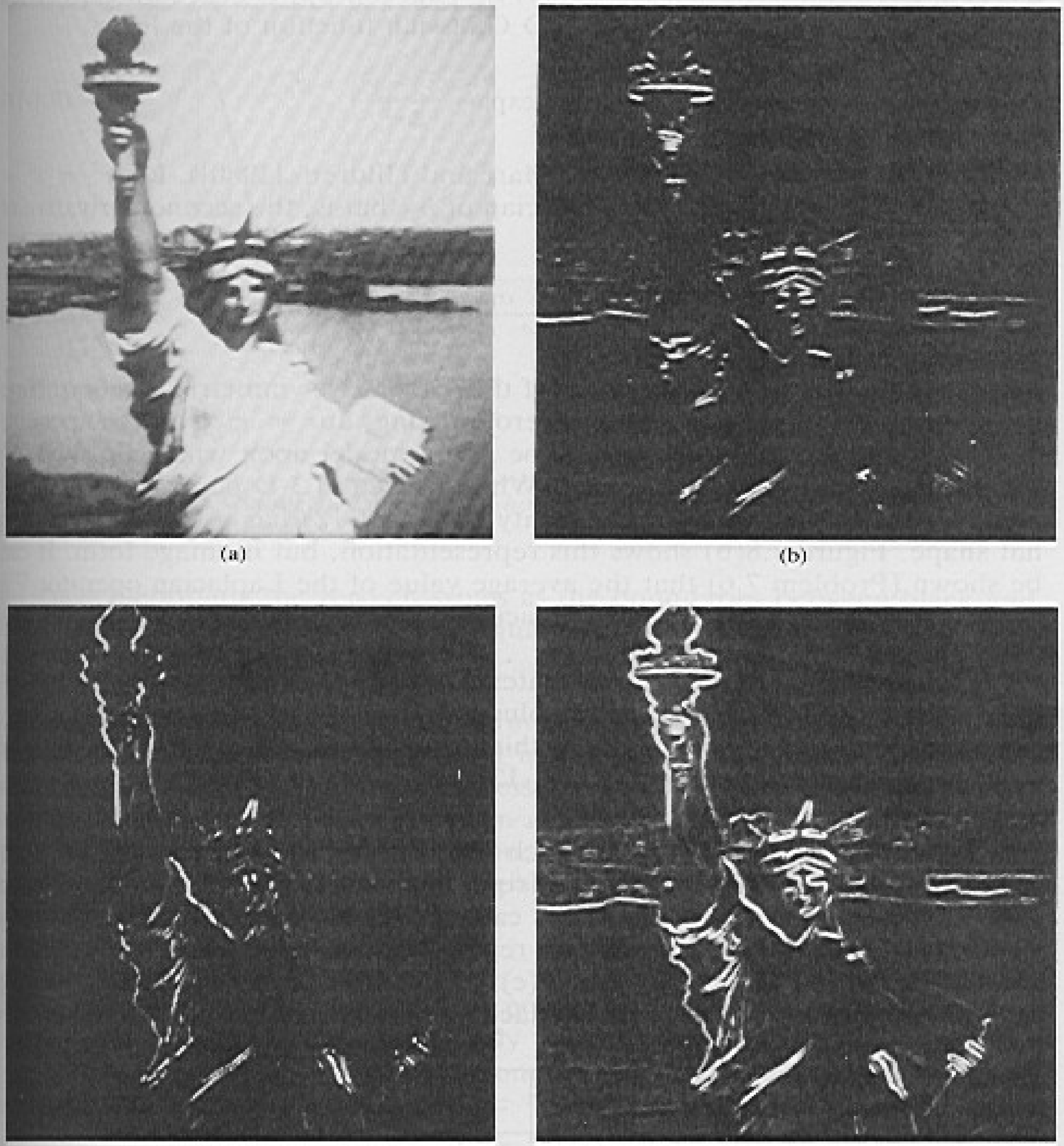
Edge Extraction via Gradients

(a) original

(b) vertical Sobel filtering

(c) horizontal Sobel filter

(d) magnitude of gradients



(a) (b) (c) (d)

Canny Edge Detector

- **Reference:** Canny, J.F., “A computational approach to edge detection,” *IEEE Trans Pattern Analysis and Machine Intelligence*, 8(6): 679-698, Nov 1986.

First Step - Smoothing

- The image is smoothed by Gaussian convolution
  - The larger the width of the Gaussian mask, the lower is the detector's sensitivity to noise.
  - The localization error in the detected edges also increases slightly as the Gaussian width is increased.

$$\frac{1}{115}\begin{array}{|c|c|c|c|c|}\hline 2 & 4 & 5 & 4 & 2 \\ \hline 4 & 9 & 12 & 9 & 4 \\ \hline 5 & 12 & 15 & 12 & 5 \\ \hline 4 & 9 & 12 & 9 & 4 \\ \hline 2 & 4 & 5 & 4 & 2 \\ \hline \end{array}$$

Discrete approximation to Gaussian function with σ=1.4

Canny Edge Detector

Second Step—Find Edge Strength

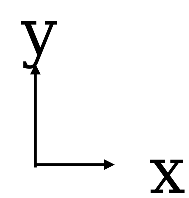
- Find the edge strength by taking the gradient of the image
- The magnitude, or EDGE STRENGTH, of the gradient is then approximated: (Norm of the gradient)

$$|G| = |Gx| + |Gy|$$

Sobel operator

-1	0	+1
-2	0	+2
-1	0	+1

Gx



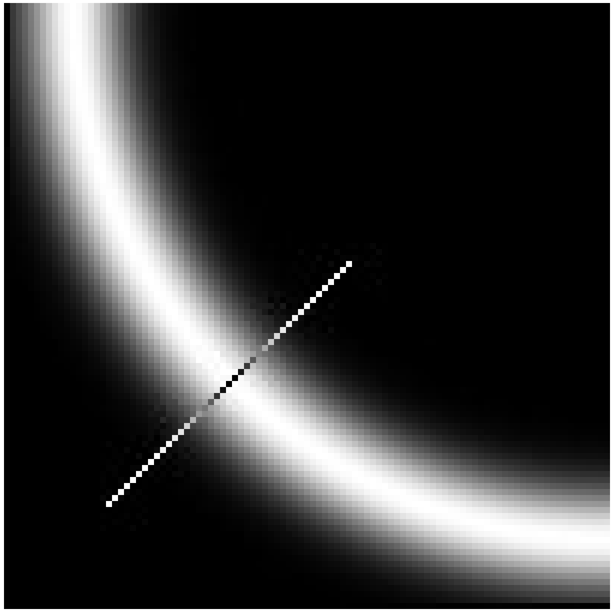
+1	+2	+1
0	0	0
-1	-2	-1

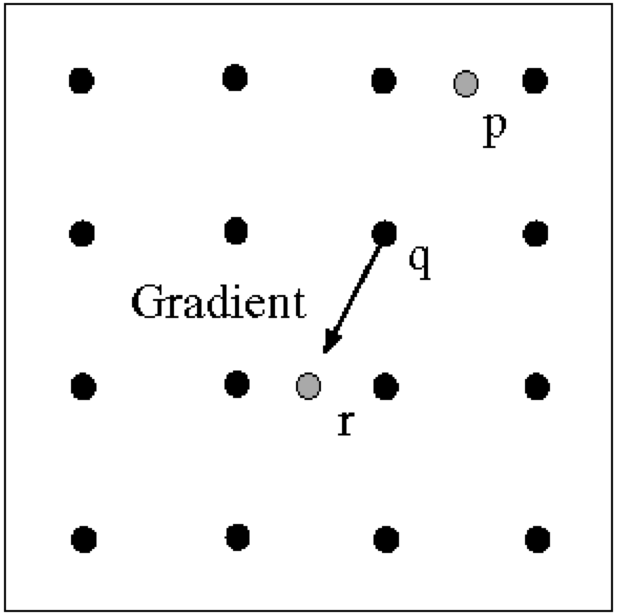
Gy

Canny Edge Detector

Third Step - Non-Maximum Suppression (Thinning)

Localize the peaks of the gradient magnitude





- Check if the pixel is a local maximum along the gradient direction
  - requires checking interpolated pixels p and r

Canny Edge Detector

Fourth Step-- Hysteresis

- If the threshold for the edges is too high, we have many broken lines; If too low, we have too many noise edges

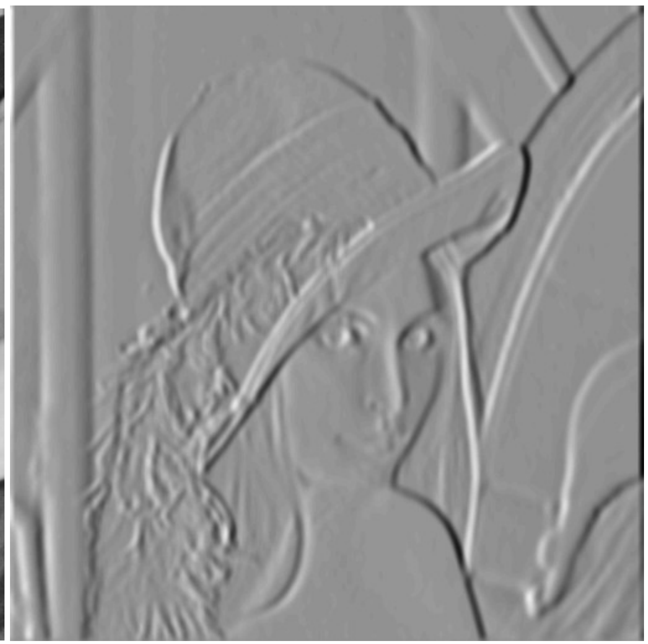
- To resolve these problems, **two thresholds** are set.

- Any pixel location that has a gradient value greater than the **high threshold** is assumed to be an edge pixel.

- Then, any pixels that are connected to this edge pixel and have a gradient value greater than the **low threshold** are also selected as edge pixels.

Canny Edge Detector - An Experiment

Original image (Lena)



Vertical gradients

Horizontal gradient

Final Result

Norm of the gradient

After thinning
(non-maximum suppression)

After Hysteresis

Effect of σ (Gaussian kernel size)

Original

Canny with $\sigma = 1$

Canny with $\sigma = 2$

- The choice of σ depends on desired behavior
  - large σ detects large scale edges
  - small σ detects fine features

Mathematical Morphology

- J. Serra, **Image Analysis and Mathematical Morphology**, Academic Press, London, 1982.
- A range of non-linear image processing techniques that deal with the shape or morphology of features in an image.
- The word morphology refers to form and structure.
- Most morphological techniques operate on binary images for

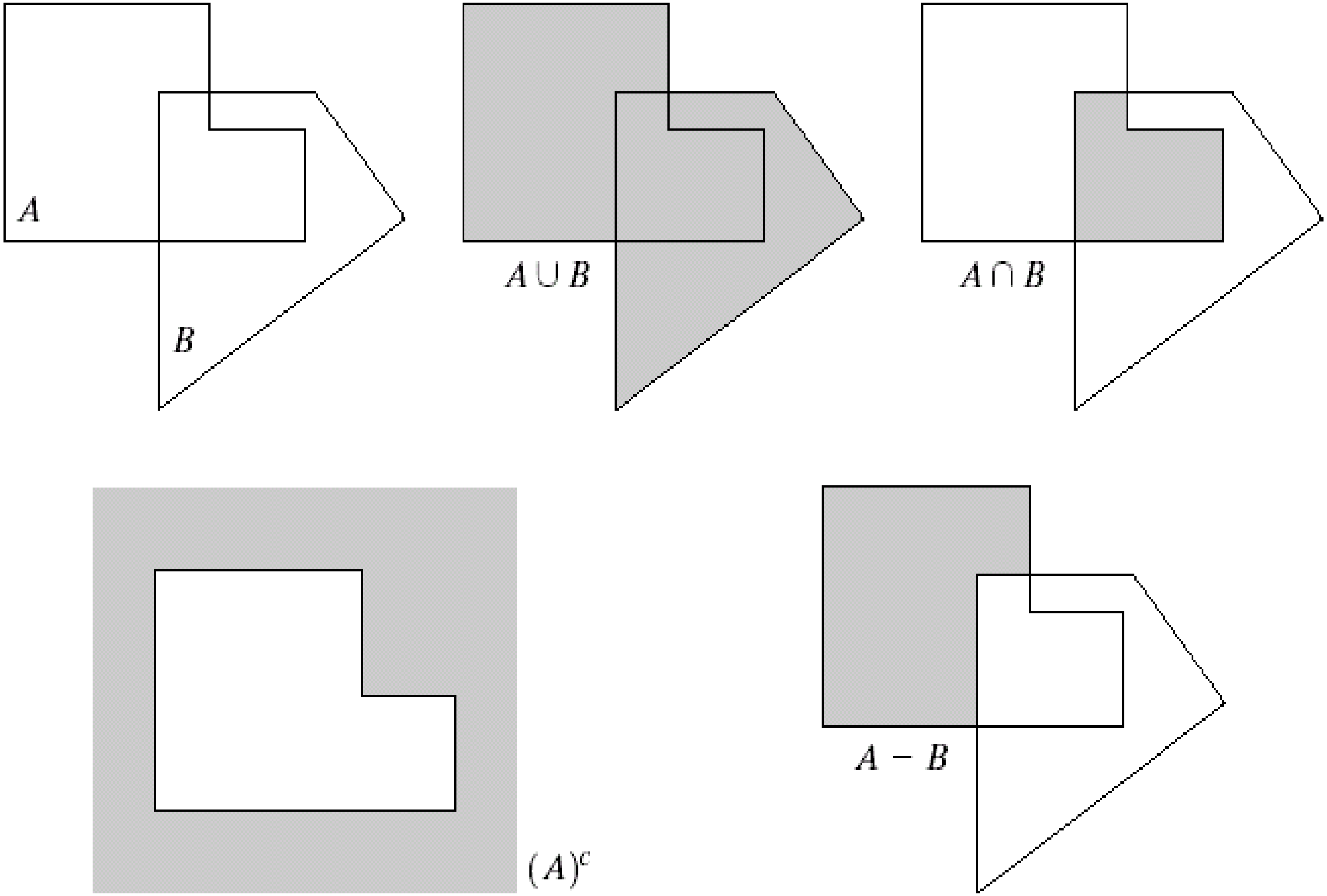
  - Noise reduction (without eliminating essential features) and feature detection

  - Analysis of connectivity of components

  - Object selection using geometric features

  - Post-processing for image segmentation

Basic Set Theory



a b c
d e

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

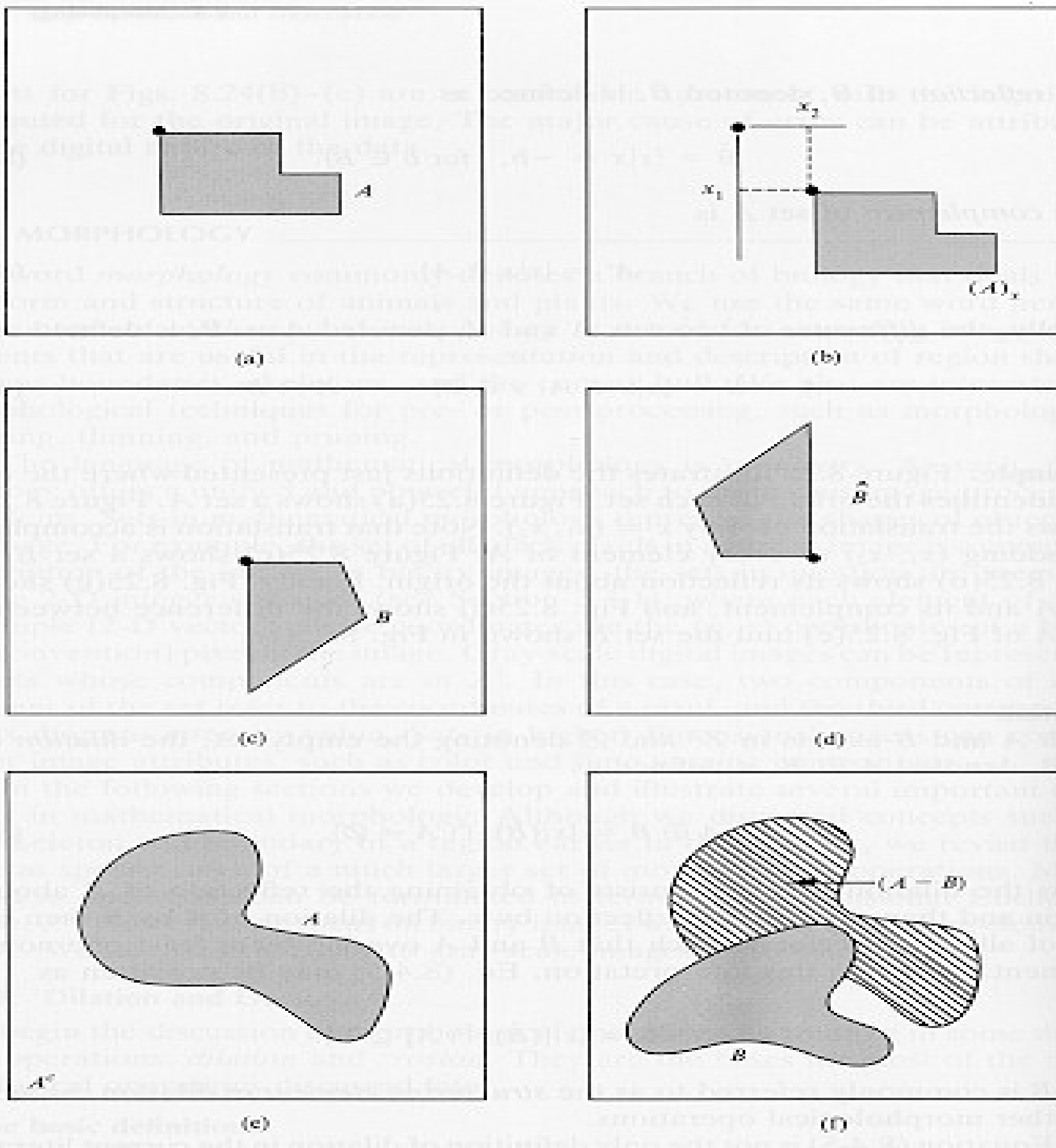
Elementary Operations

Translation: $(A)_z = \{x \mid x = a + z, \text{ for } a \in A\}$

Reflection: $\hat{A} = \{x \mid x = -a, \text{ for } a \in A\}$

Complement: $A^c = \{x \mid x \notin A\}$

Difference: $A - B = \{x \mid x \in A, x \notin B\} = A \cap B^c$



A
x_1
x_2
$(A)_x$
(a)
(b)
B
$\hat{B}$
(c)
(d)
A
A^c
$(A - B)$
B
(e)
(f)

Structuring Elements

- Morphological operations use a small shape or template known as a structuring element (SE).
- The structuring element is positioned at all possible locations in the image and is compared to the corresponding neighborhood of pixels.
- Morphological operations differ in how they carry out this comparison. Some test whether the SE "fits" within the neighborhood, others test whether it "hits" or intersects the neighborhood.

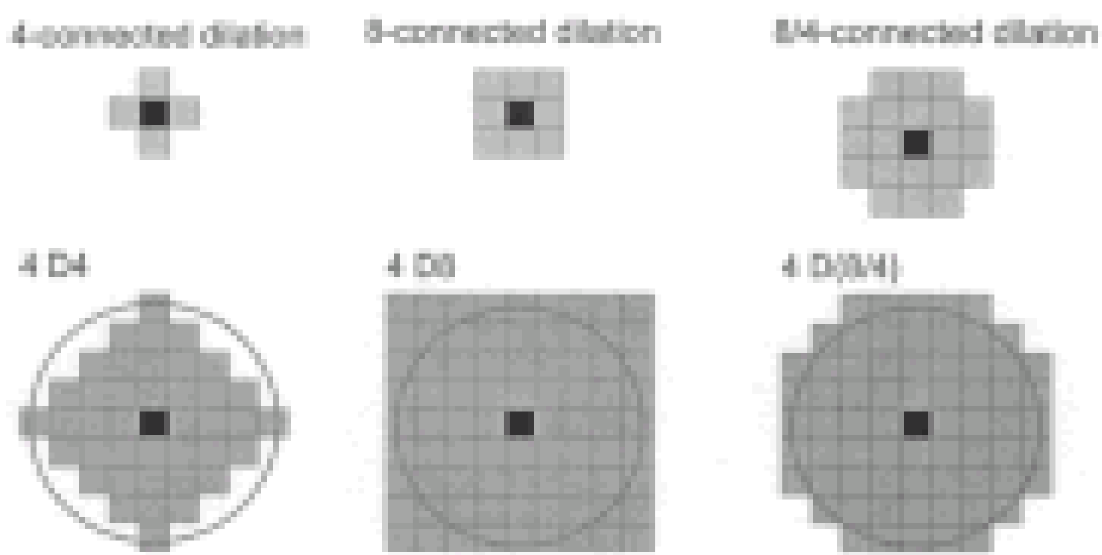
Structuring Elements

- The structuring element applied to a binary image can be represented as a small matrix of pixels, each with a value of 1 or 0.
- The dimensions of the matrix determine the size of the SE, and its shape is determined by the pattern of ones and

$$SE = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Characteristics of an SE

- A structuring element has an origin (just like convolution mask).
- A SE is analogous to the kernel in convolution.
- The shape and size of the SE must be adapted to the geometric properties of the image objects of interest, i.e., an SE takes into account a number of factors: shape, size, and orientation.
- Disk SE examples



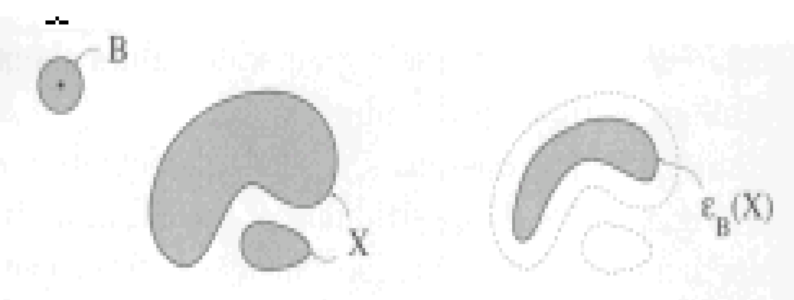
Basic Morphological Operations

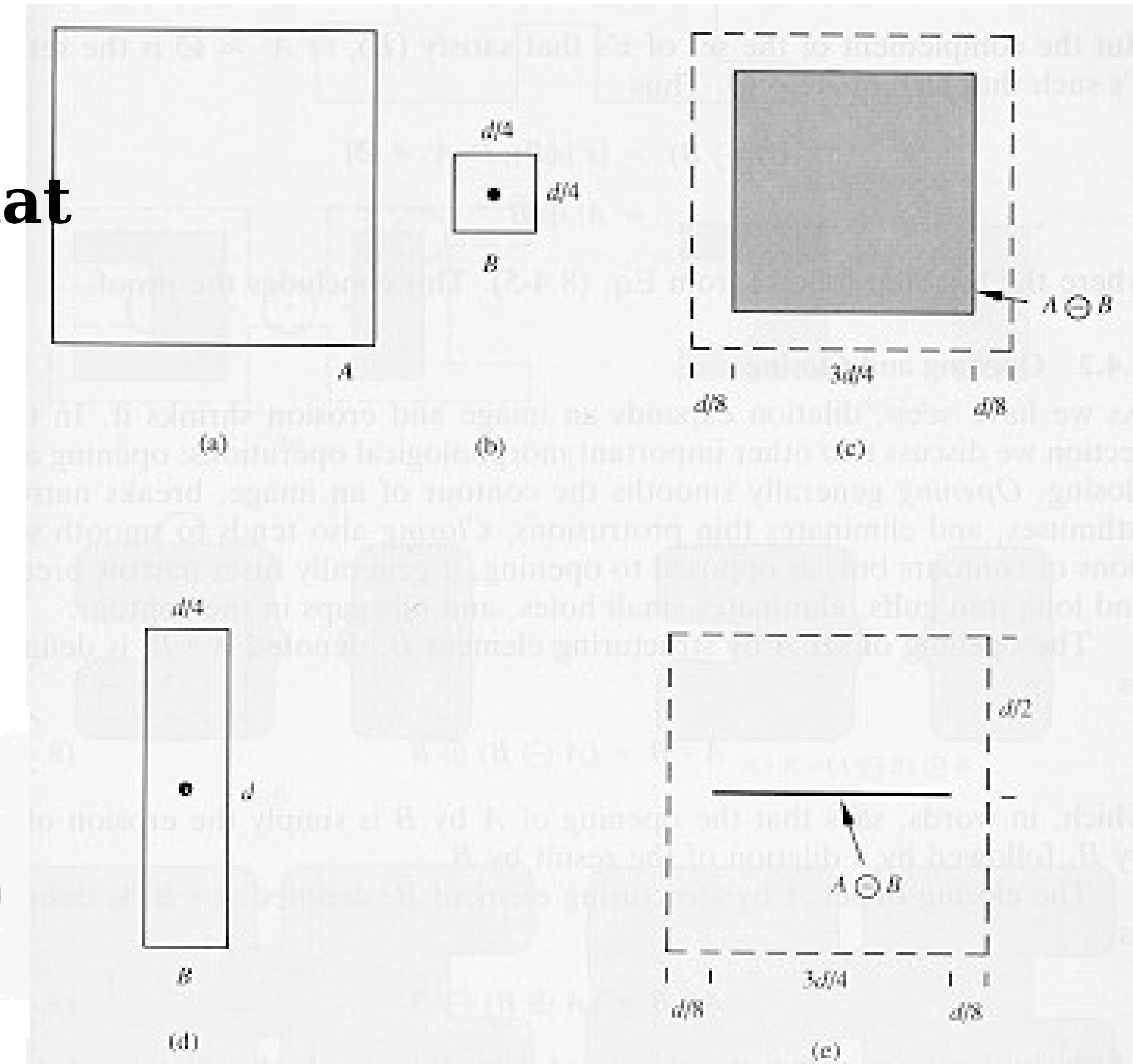
- Erosion: shrinks or erodes a region:
  - It expands the holes enclosed by a single region and make the gaps between different regions larger.
- Dilation: expands or dilates a region
  - It shrinks the holes enclosed by a single region and make the gaps between different regions smaller.

Erosion

he set of all x such that
B translated by x” is
ontained in A.

$$A \Theta B = \{x | (B)_x \subseteq A\}$$

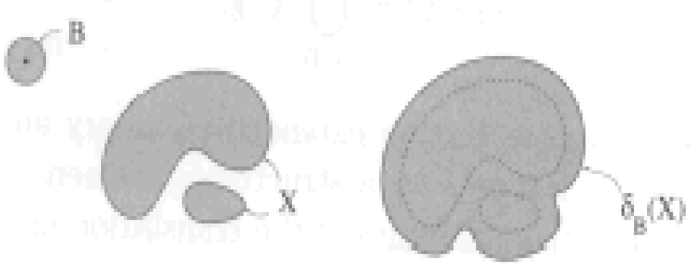




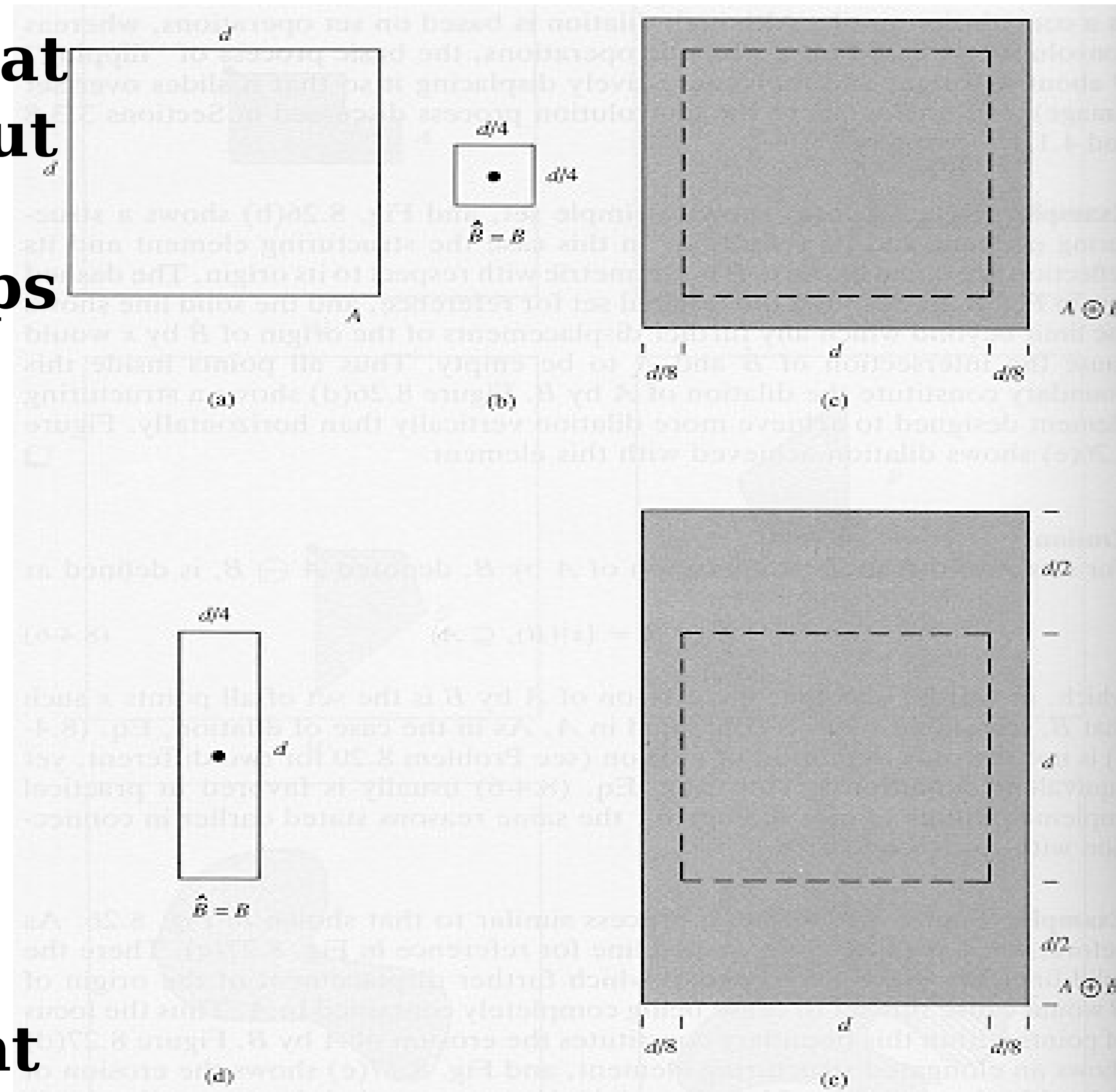
Dilation

he set of all x such that he reflection of B about s origin and then ranslated by x, overlaps vith A by at least one onzero element.

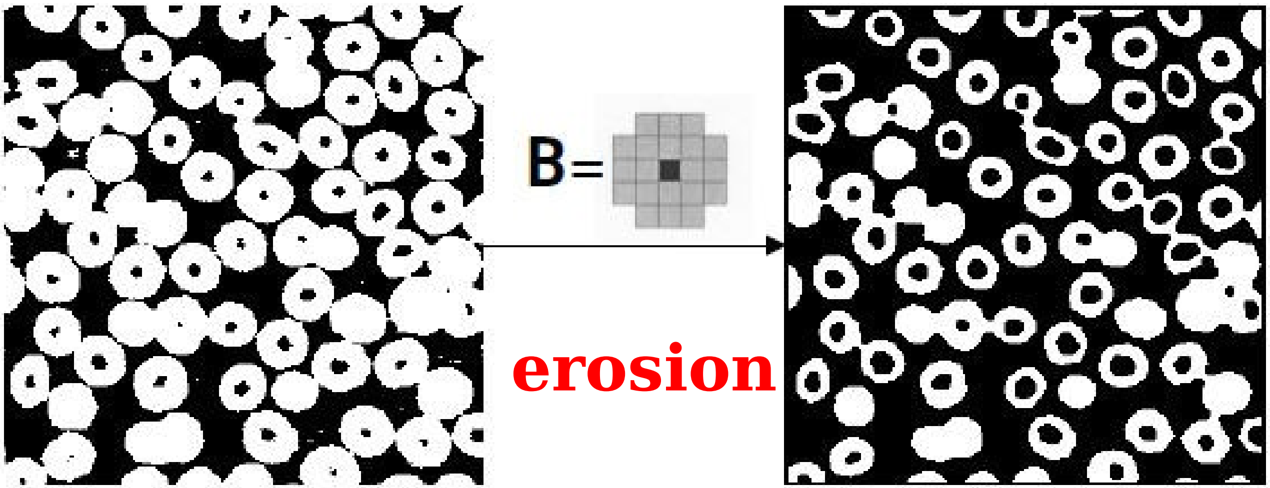
$$A \oplus B = \{x \mid (\hat{B})_x \cap A \neq \emptyset\}$$

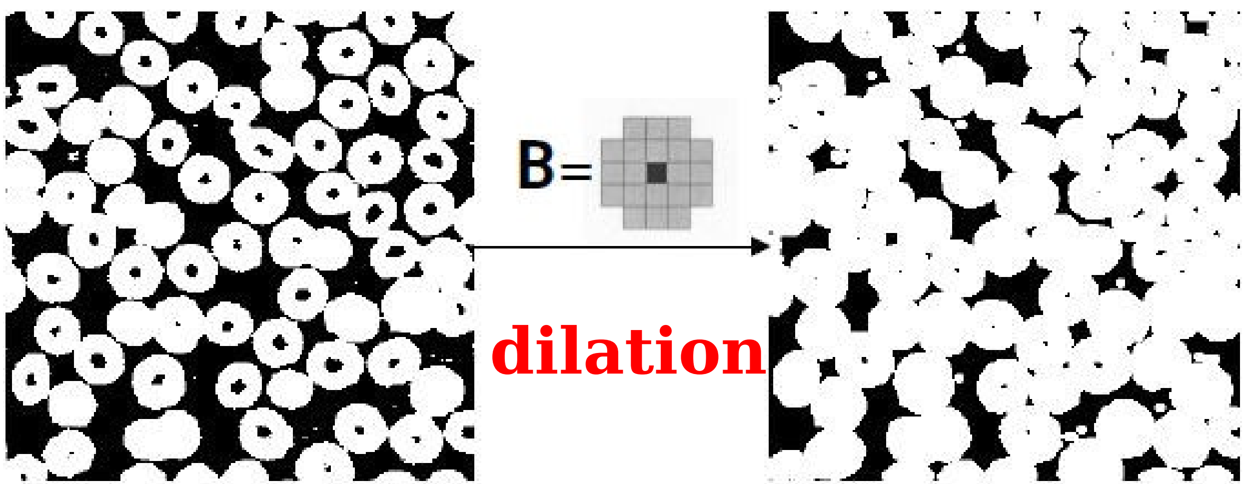


B: structuring element

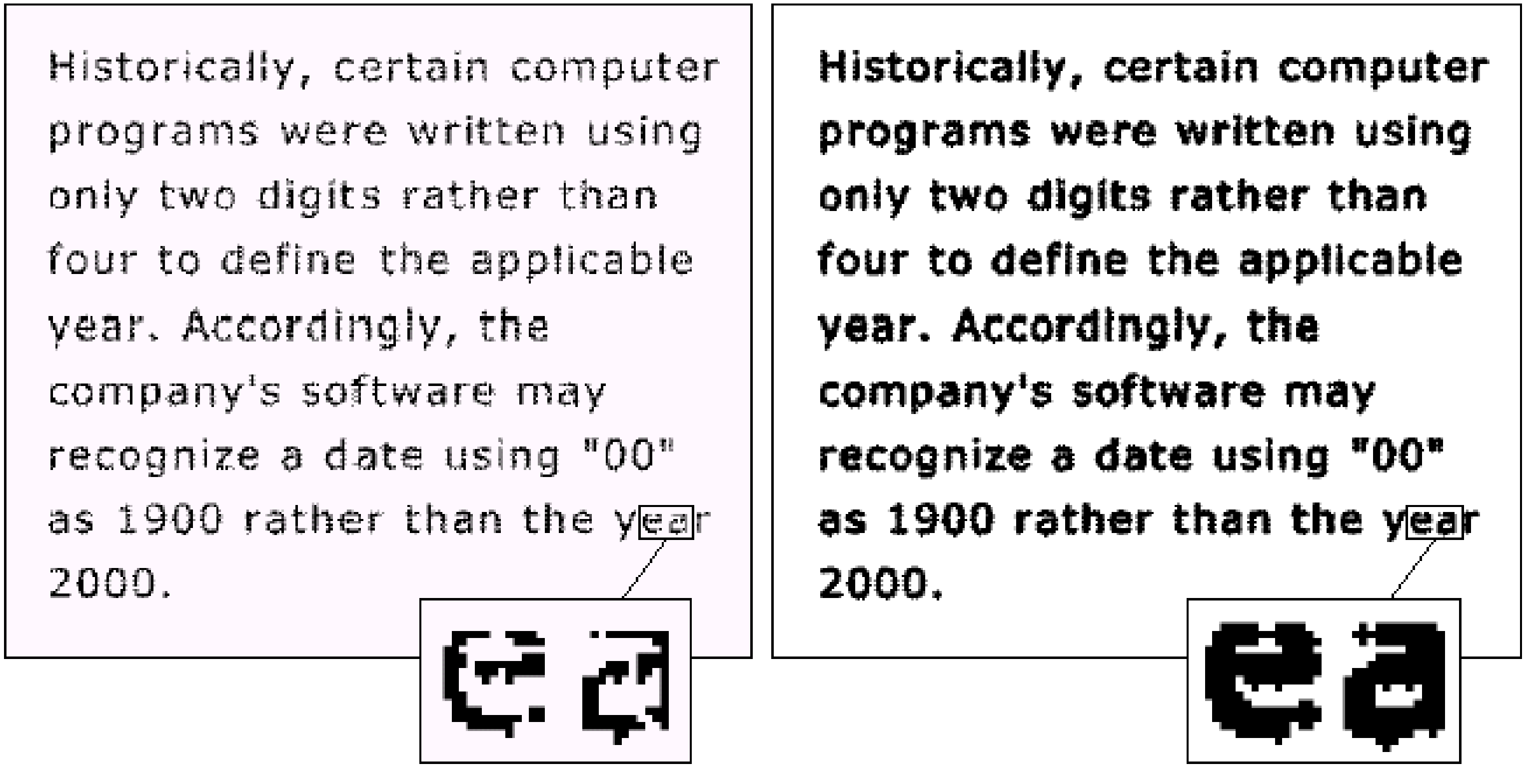


Erosion and Dilation





Dilation



0	1	0
1	1	1
0	1	0

a c
b

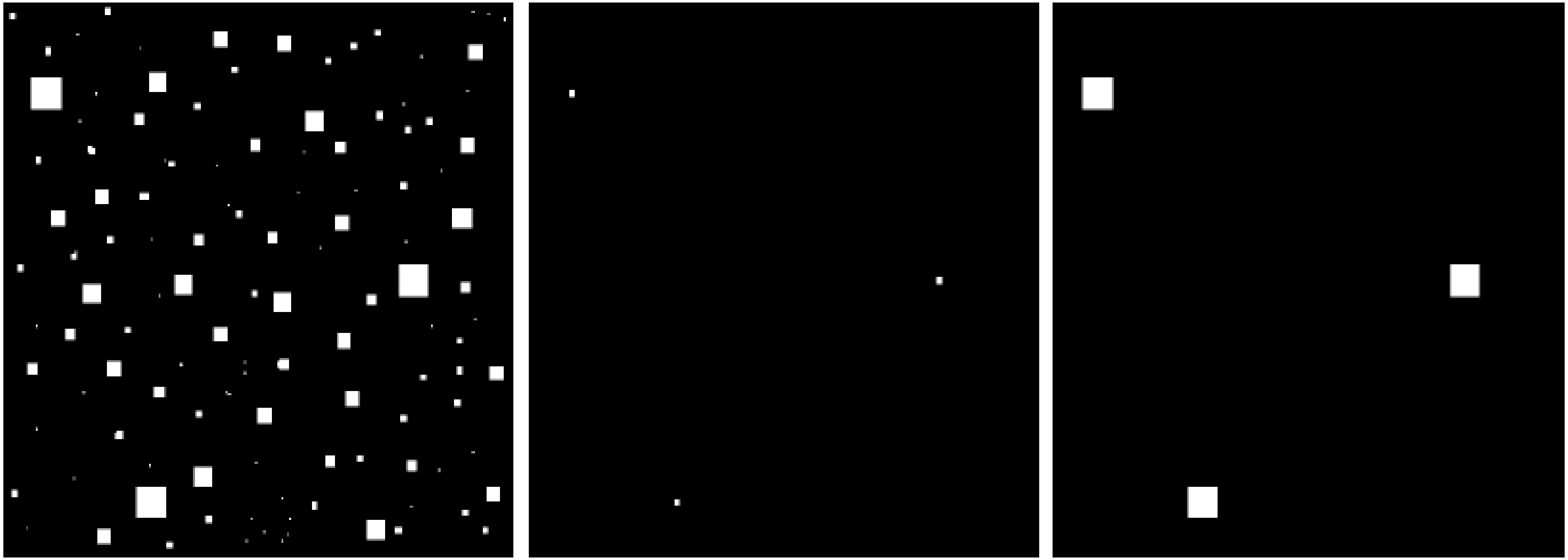
FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Compound Morphological Operations

Combinations of the elementary operations of erosion and dilation:

- **Opening:** use of erosion, followed by dilation.
  - The effect is to smooth boundaries, to break narrow isthmuses, and to eliminate small noise regions.
  - Separate connected objects, remove small objects
- **Closing:** use of dilation, followed by erosion.
  - The effect is to smooth boundaries, to join narrow breaks, and to fill small holes caused by noise.
  - Connect disconnected objects

Opening



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Opening

$$A \circ B = (A \ominus B) \oplus B$$

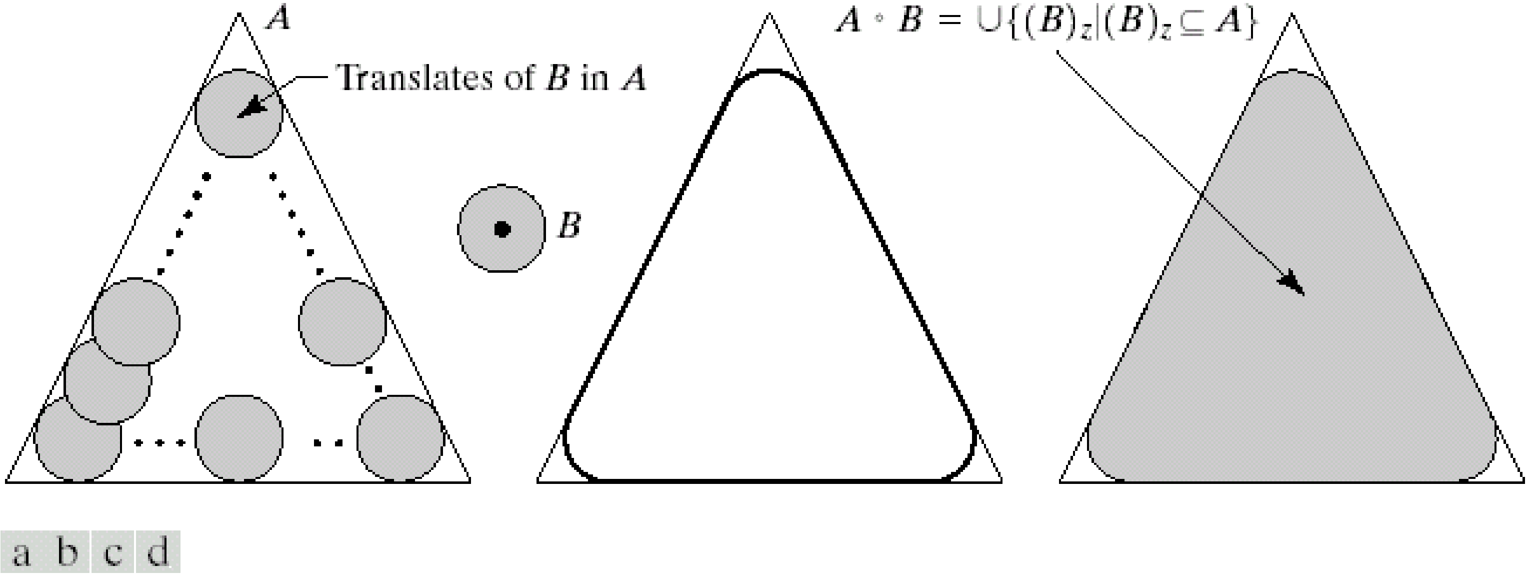
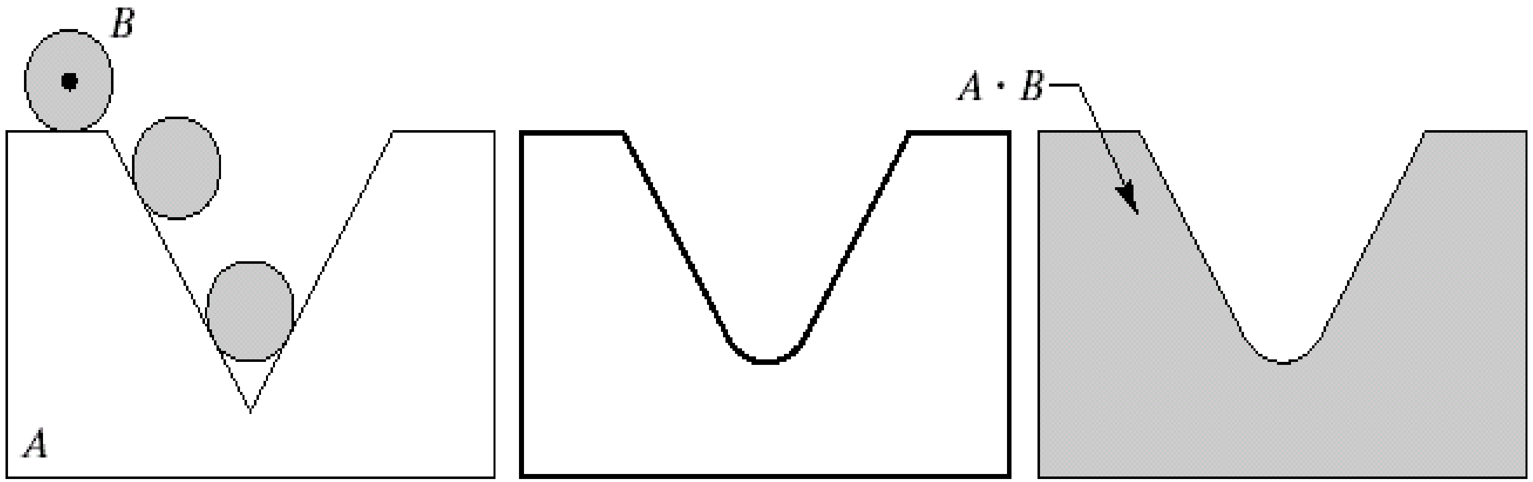


FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Closing

$$A \bullet B = (A \oplus B) \ominus B$$



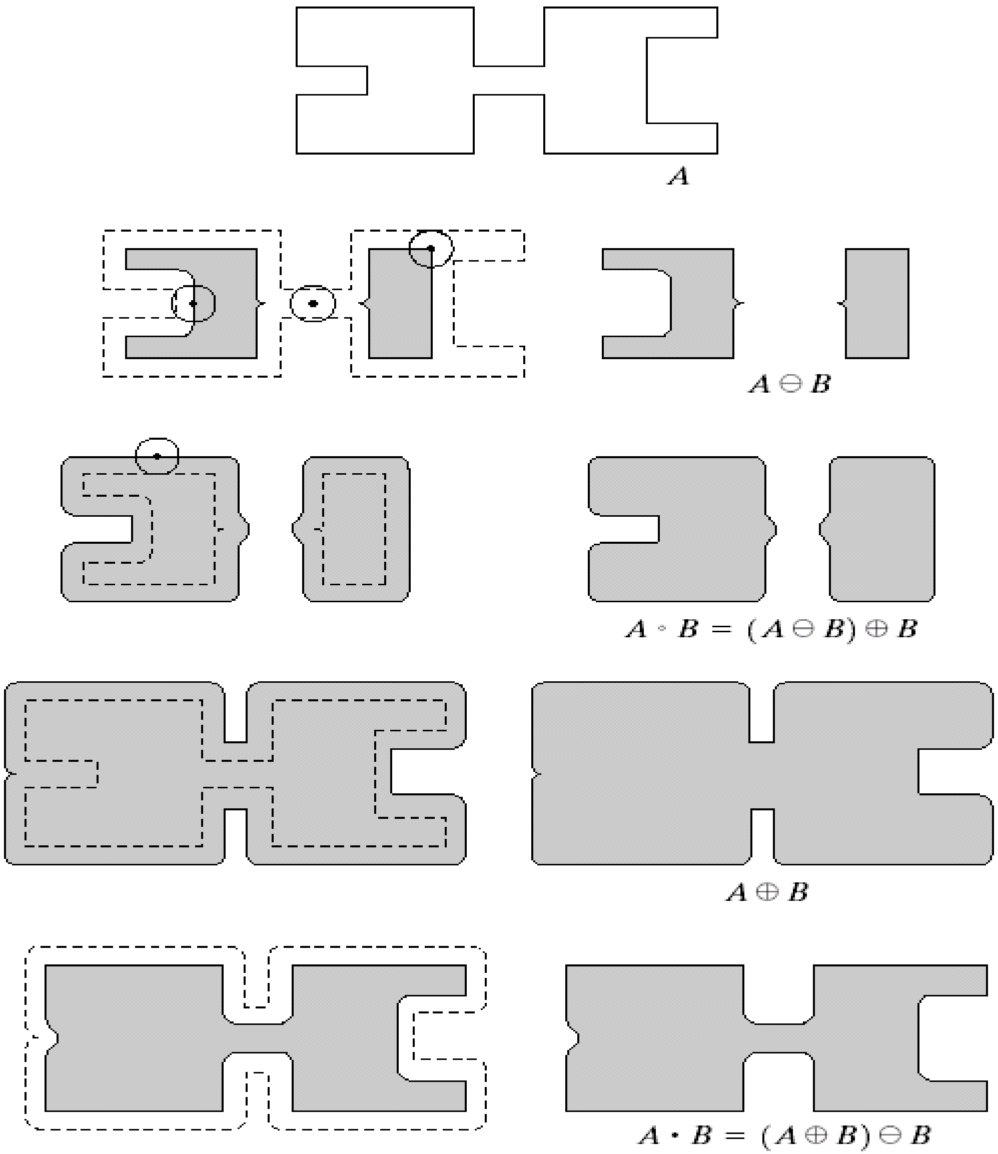
a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

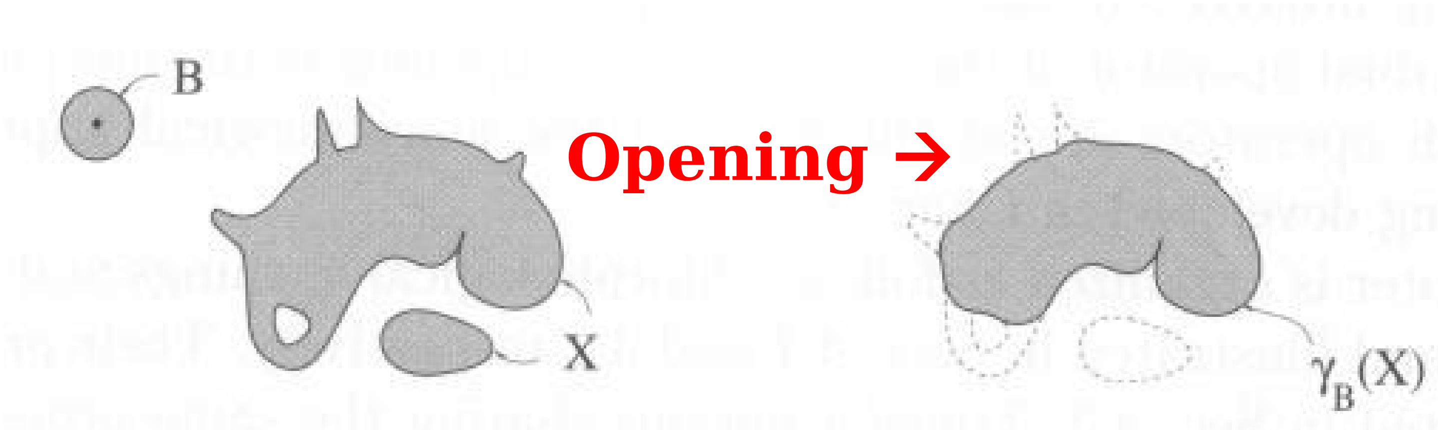
Opening and Closing

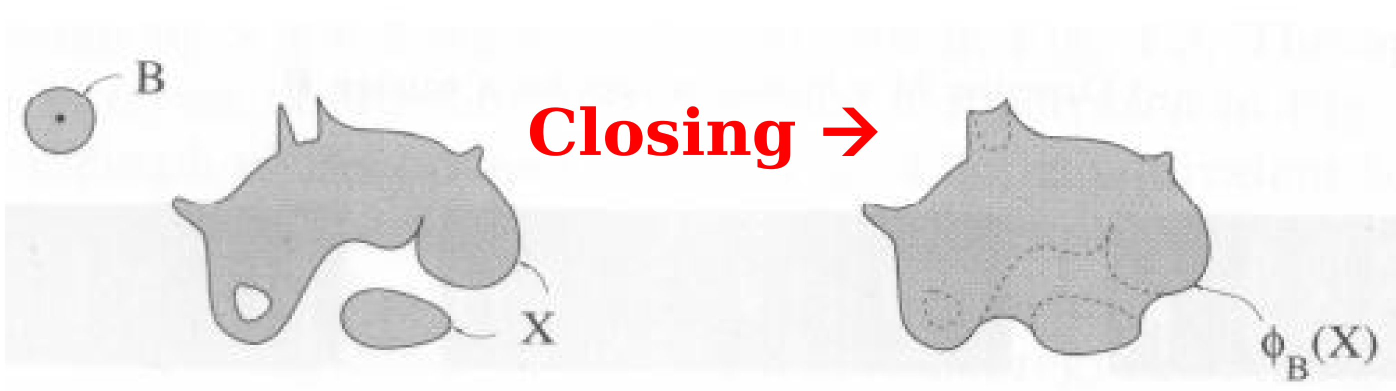
a	
b	c
d	e
f	g
h	i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

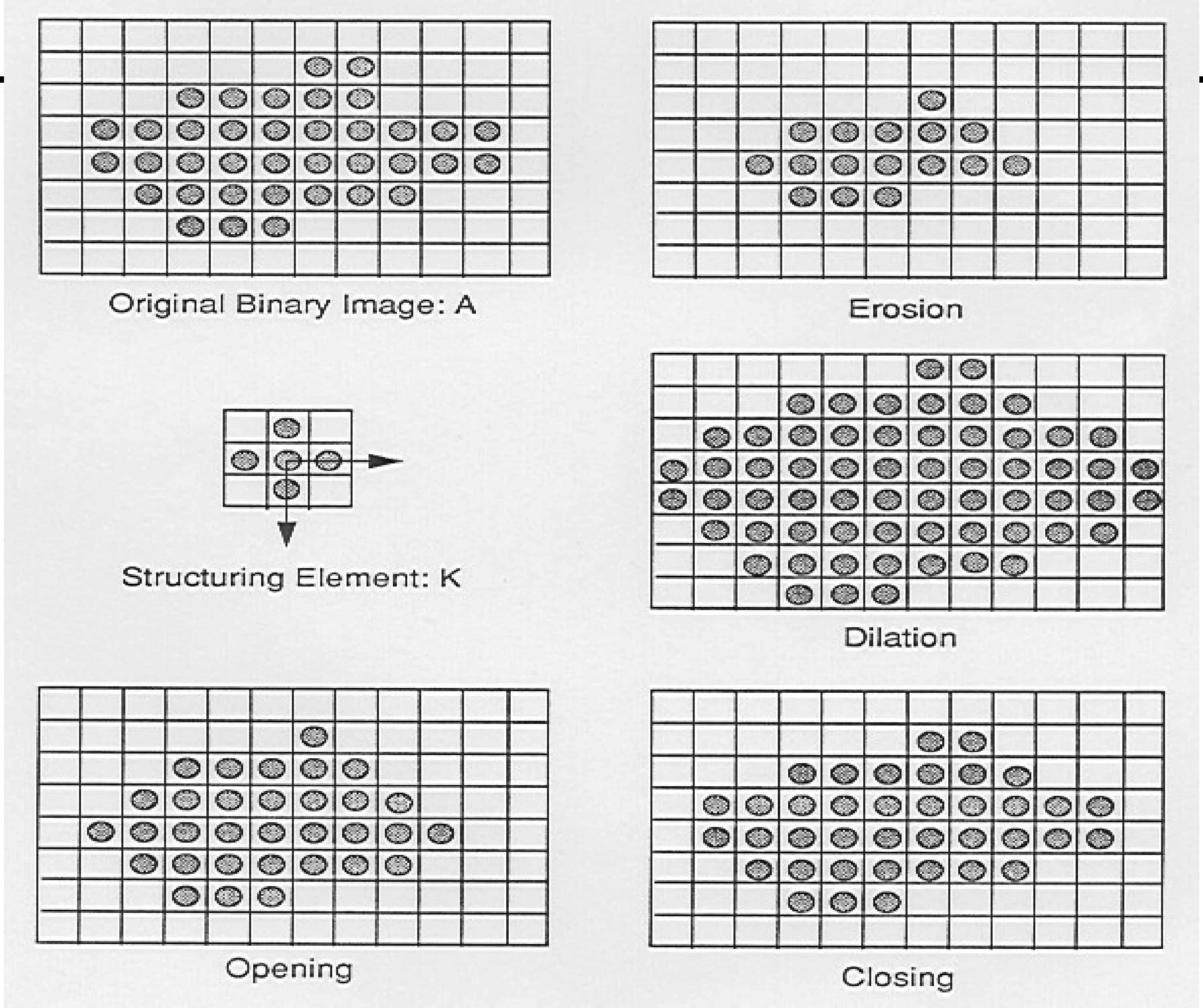


Opening and Closing





Basic Morphological Operations

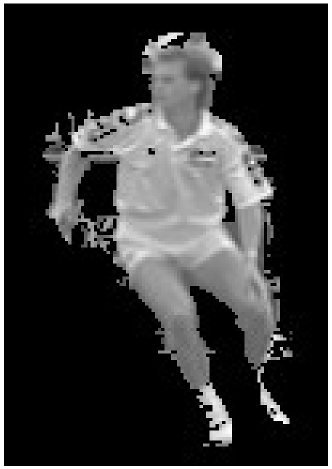


Opening vs. Closing

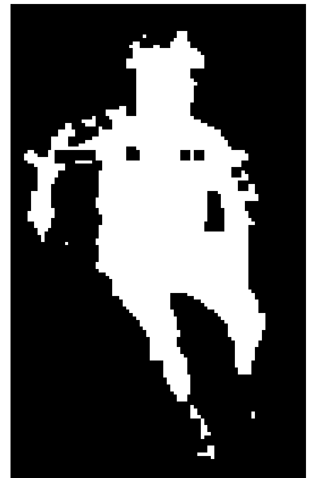
– Opening is used when the image has small regions to be removed. It is not used for narrow regions where there is a chance that the initial erosion operation might unintentionally disconnect the regions.

– Closing is used when a region has become disconnected and the desire is to restore the connectivity. It is not used when different regions are located closely and the initial dilation might connect them.

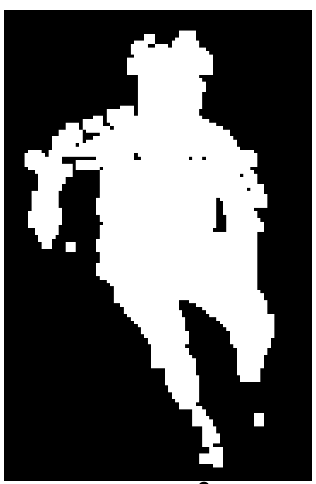
Example (using a 3x3 structuring element)


motion detection


binary

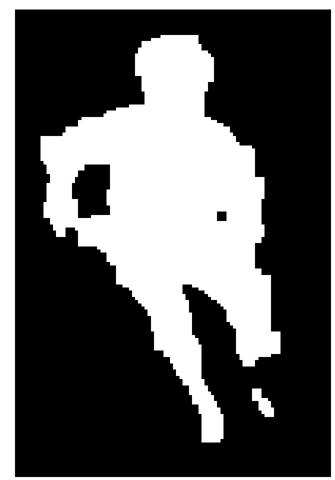

erosion


dilation

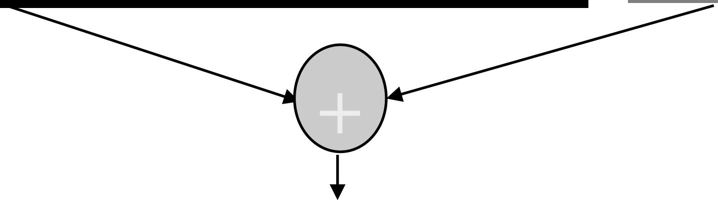

opening


closing


open-close

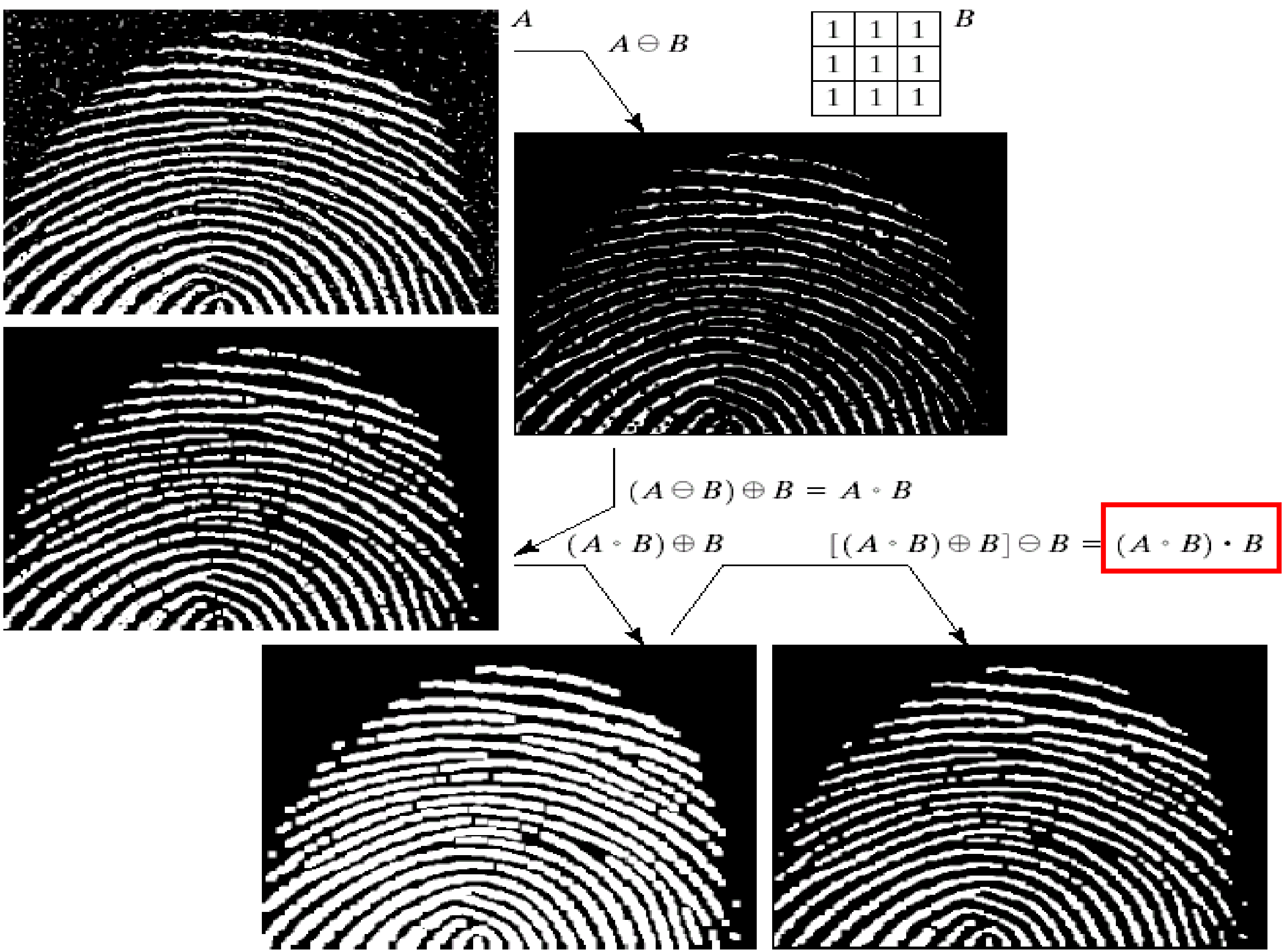

close-open

Segmentation Application Example



Sprite coding

Fingerprint Image Enhancement



a b
d c
e f

FIGURE 9.11
(a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Some Basic Morphological Algorithms

- Hit-or-Miss Transformation
- Boundary (Contour) Extraction
- Region Filling
- Extraction of Connected Components
- Convex Hull
- Thinning
- Thickening
- Skeletonization
- Pruning (reduces short branches extruding from a region after skeletonization).

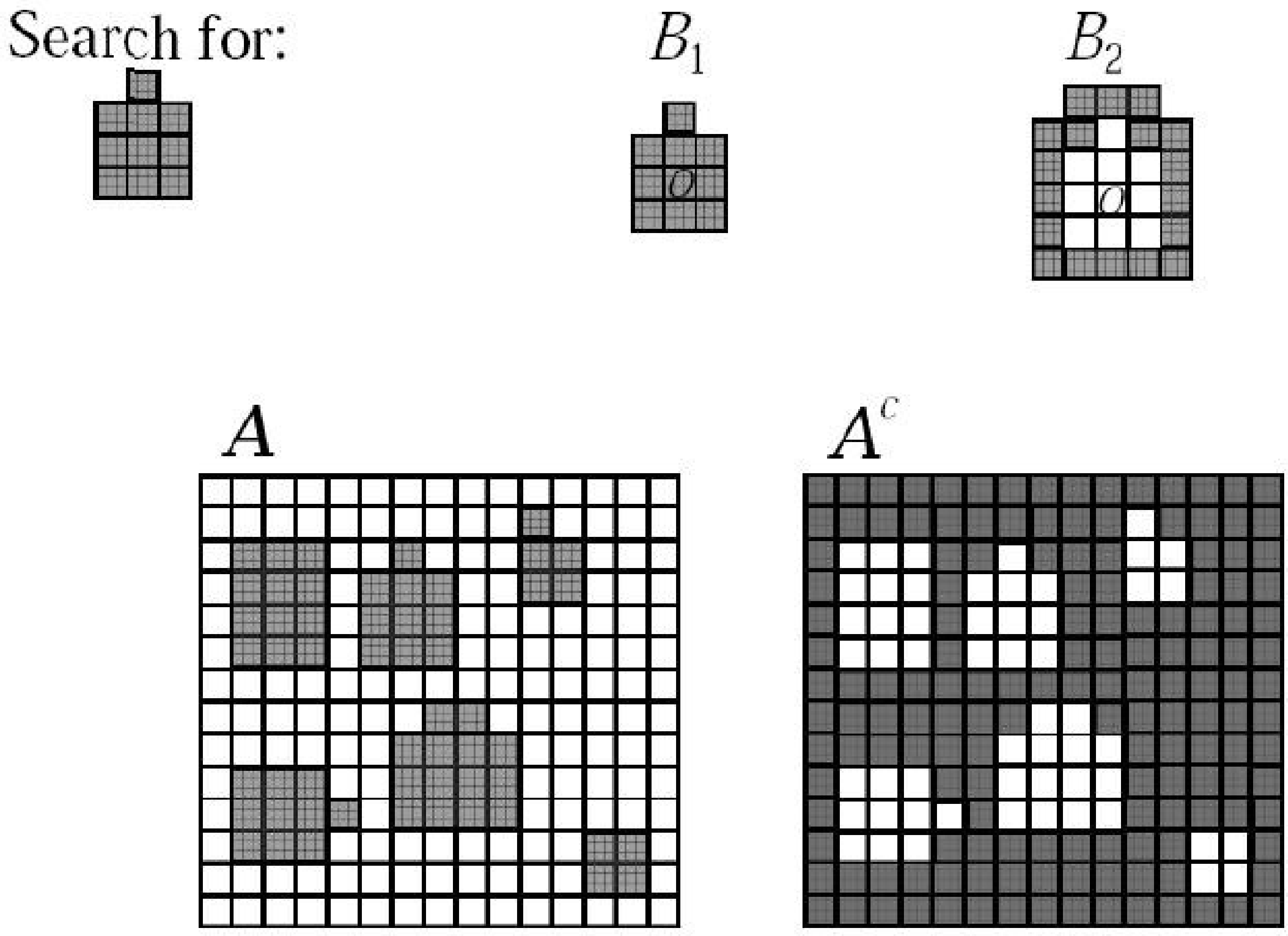
Hit-or-Miss Transform

- Serves to detect features in the image that match the shape of the structuring element.
- Requires a matched pair of SEs {**B1**,**B2**} that probe the inside and outside of objects in the image respec

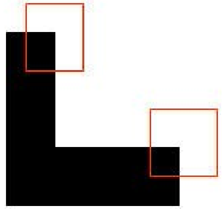
$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

$$= (A \ominus B_1) - (A \oplus \hat{B}_2)$$

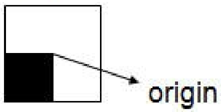
Hit-or-Miss Transform



Example of
Hit-or-Miss



X

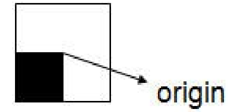


Template B

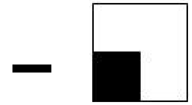
How to find the match of A in X using a computer?

Hit: the southwest quadrant must be black in X
Hit: the other three quadrants must be white in X

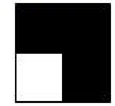
Hit: the southwest quadrant must be black in X
Hit: the other three quadrants must be black in X^c



Template B



Template B_1



Template B_2

Hit: the southwest quadrant must be black in X

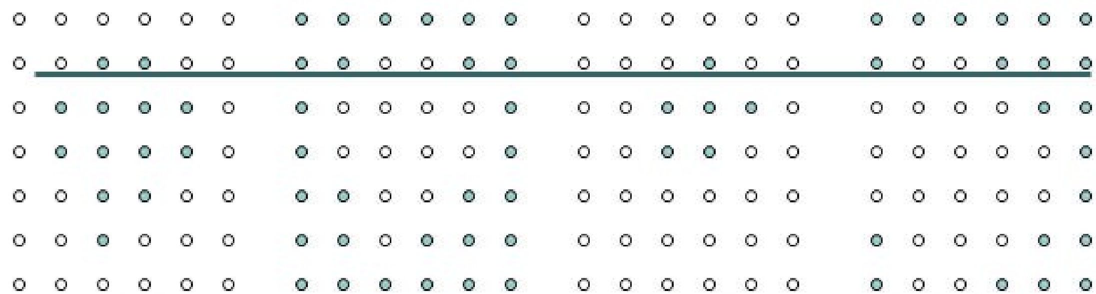
$$X_1 = X \ominus B_1$$

Hit: the other three quadrants must be black in X^c

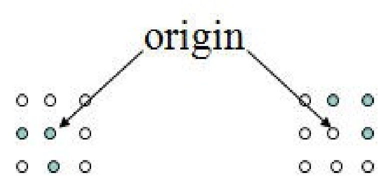
$$X_2 = X^c \ominus B_2$$

To satisfy both conditions, we need to take the intersection of X_1 and X_2

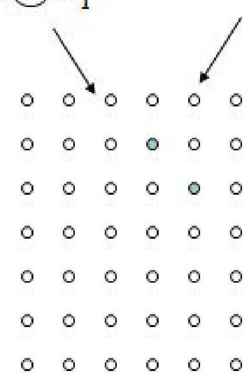
Example 1



X X^c $X \ominus B_1$ $X^c \ominus B_2$

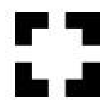
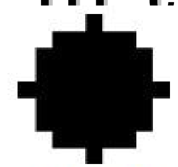


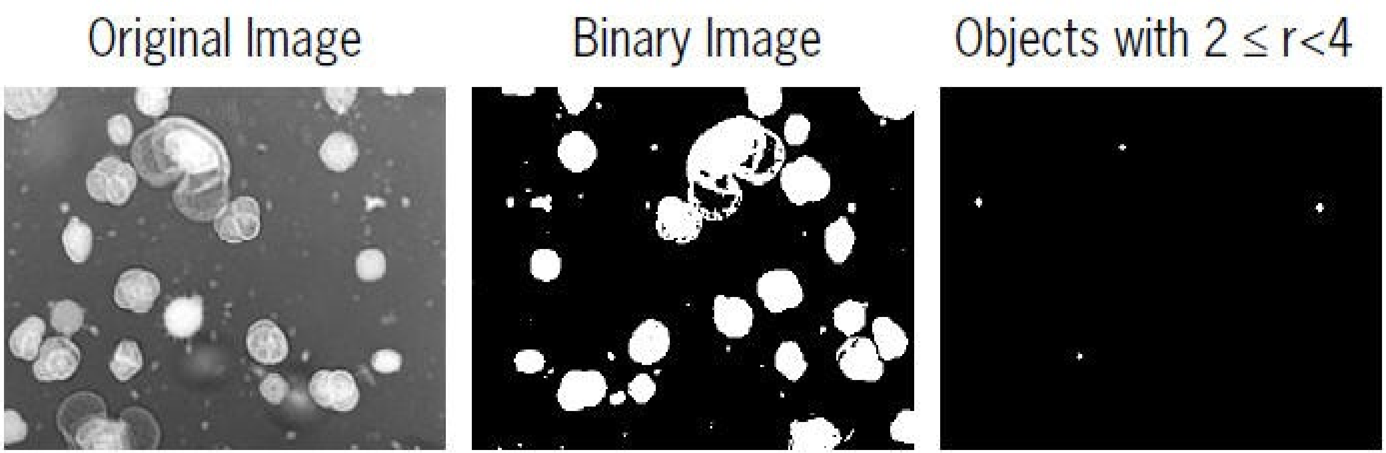
mask B_1 mask B_2



$X \circledast B$

Hit-or-Miss Transform

- Identify blobs with a radius of at least 2, but less than 4 in the pollen image. These regions totally enclose a disc of radius 2, contained in the 5 x 5 kernel named "hit", and in turn, fit within a hole of rac   co l "m

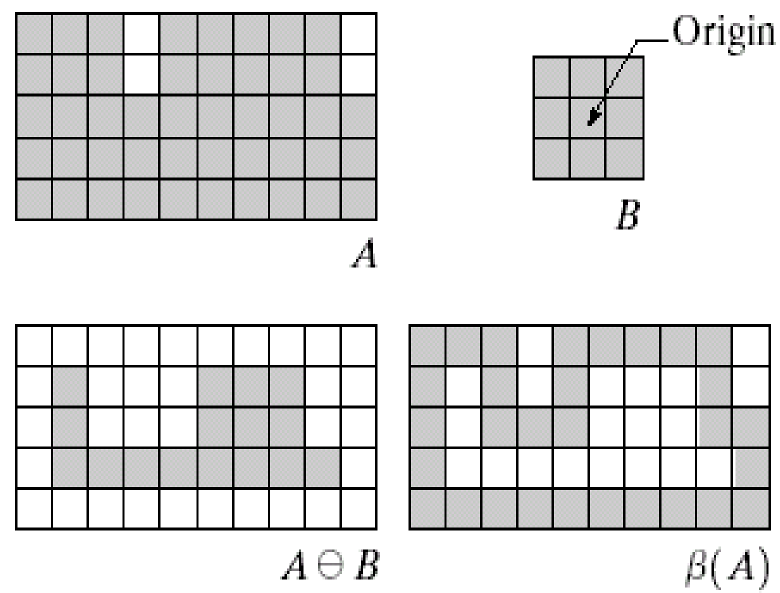


Boundary (Contour) Extraction

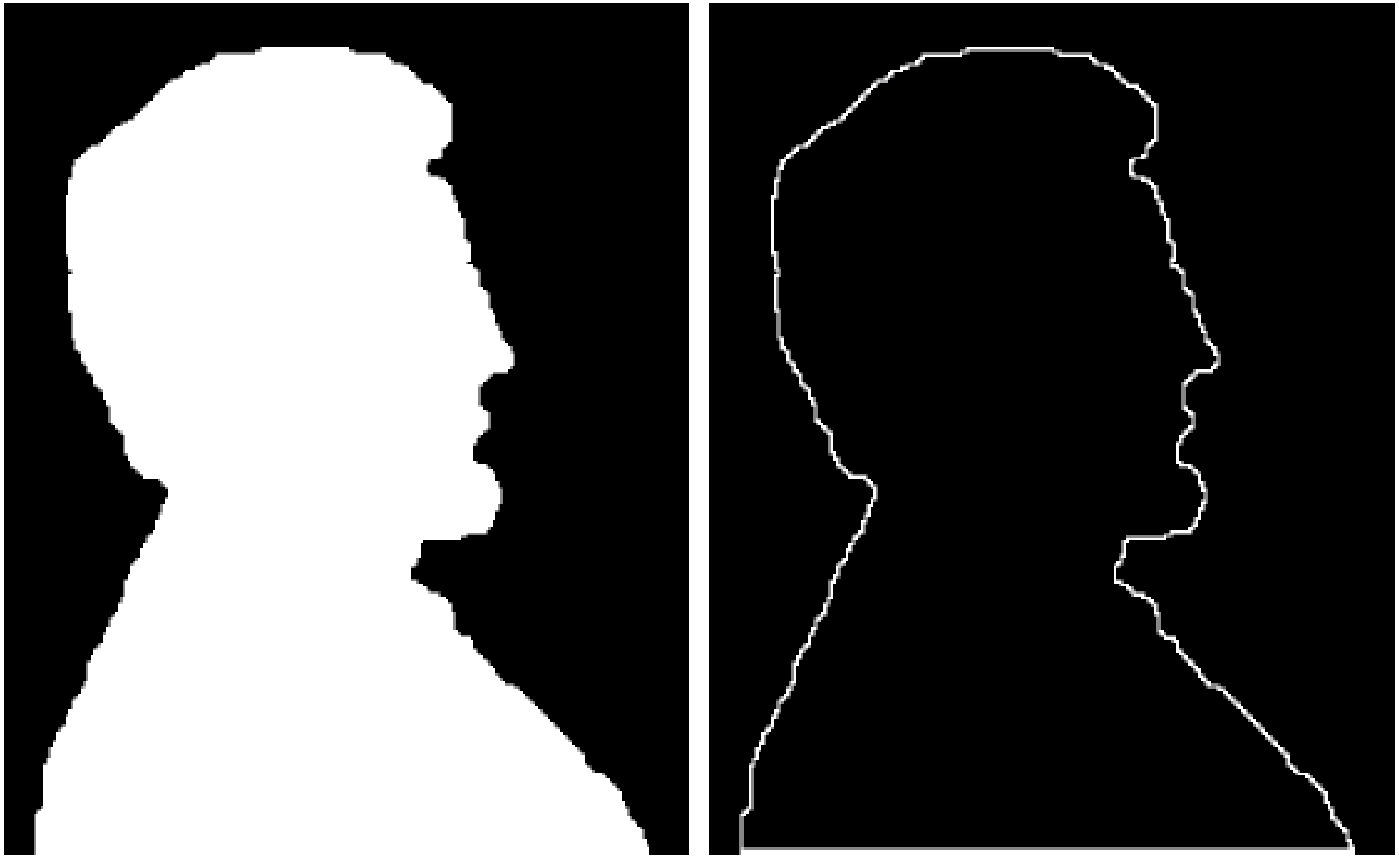
- One application of erosion is contour extraction – the contours are extracted by subtracting the eroded image fr $\beta(A) = A - (A \ominus B)$ al

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



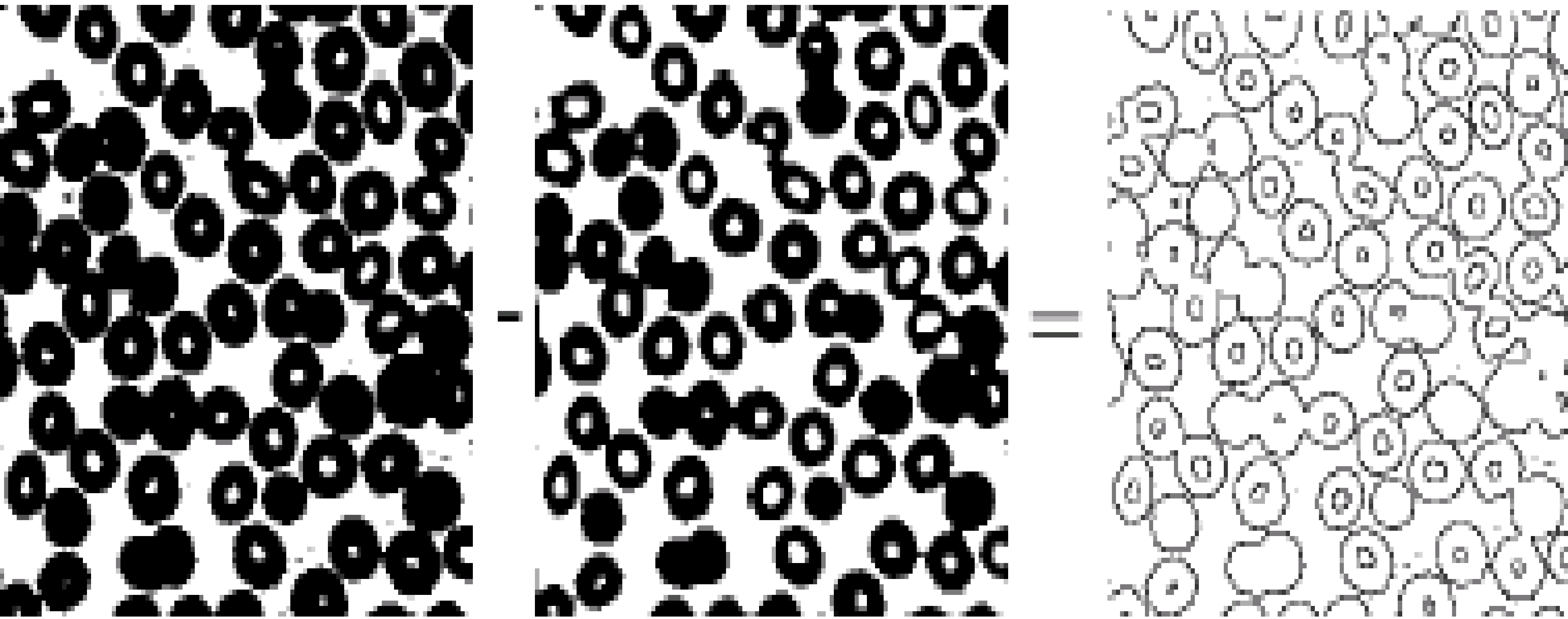
Boundary Extraction Example



a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Boundary Extraction Example



Application of Skeletonization

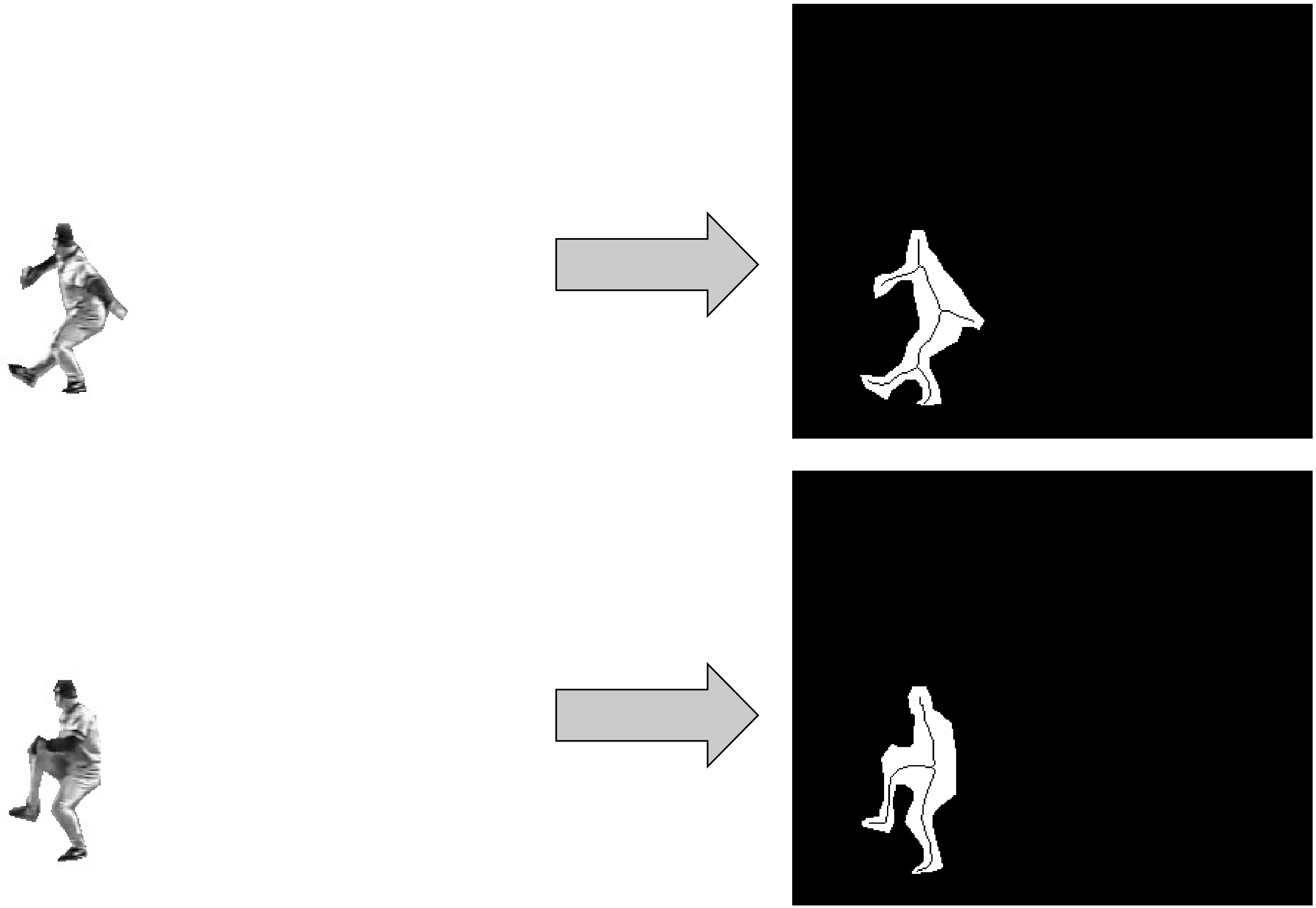


TABLE 9.2
Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$.	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

TABLE 9.2
Summary of morphological results and their properties. *(continued)*

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.

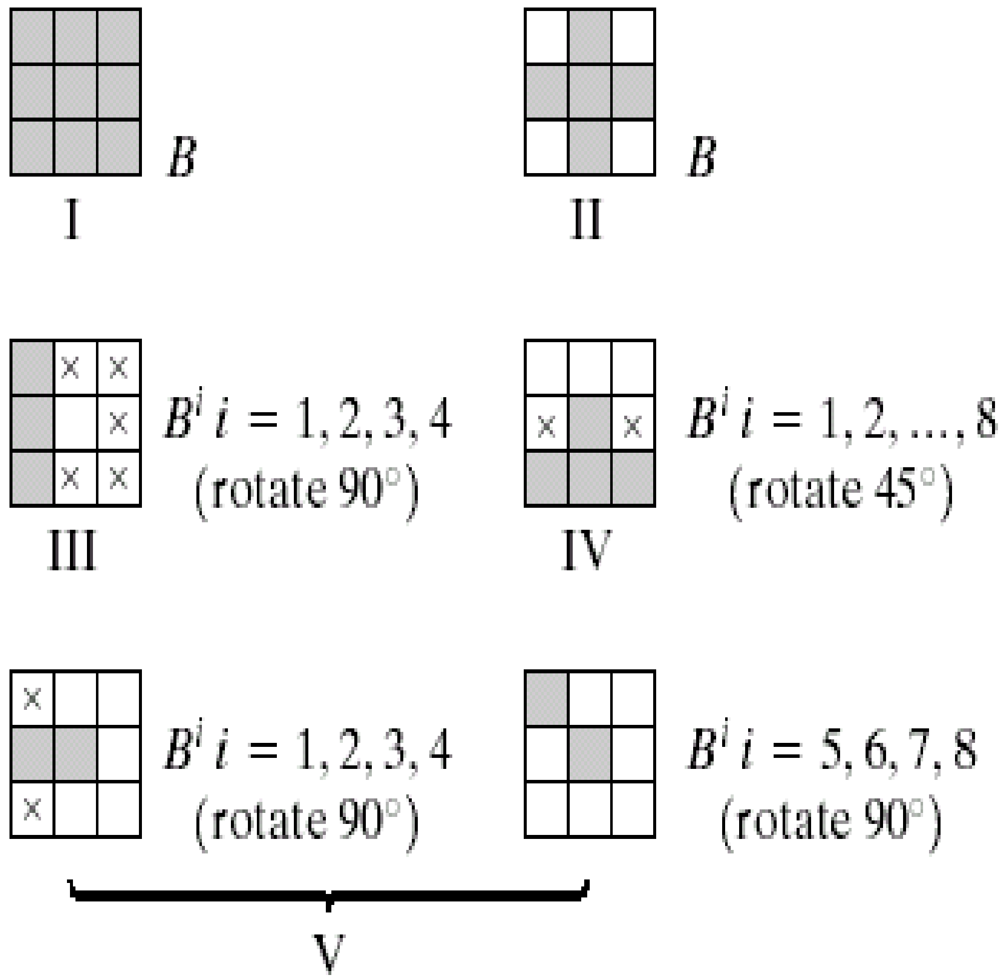


FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.